Name .... ID .... Section ...... Seat No ......

Sirindhorn International Institute of Technology
Thammasat University

Midterm Exam Answers: Semester 2, 2013

Course Title: CSS322 Security and Cryptography

Instructor: Steven Gordon

Date/Time: Thursday 9 January 2014; 13:30–16:30

Instructions:

- This examination paper has 18 pages (including this page).
- Conditions of Examination: Closed book; No dictionary; Non-programmable calculator is allowed
- Students are not allowed to be out of the exam room during examination. Going to the restroom may result in score deduction.
- Turn off all communication devices (mobile phone etc.) and leave them at the front of the examination room.
- The examination paper is not allowed to be taken out of the examination room. A violation may result in score deduction.
- Write your name, student ID, section, and seat number clearly on the front page of the exam, and on any separate sheets (if they exist).
- Reference material included at the end of the exam may be used.

Prepared by Steven Gordon on 15 January 2014
css322y13s2e01, Steve/Courses/2013/s2/css322/assessment/midterm-exam.tex, r3081

Question 1 [13 marks]

You have an RSA key pair of ($PU = \{7, 527\}, PR = \{343, 527\}$). You also know Steve and Thanaruk's public keys:

- Steve: $\{3, 319\}$
- Thanaruk: $\{7, 589\}$

You have a message, $M = 10$, to send confidentially to Steve.

(a) What is the value of the ciphertext that you send to Steve? [3 marks]

Answer: ____

Answer. *To send to Steve, you must use Steve's public key to encrypt.*

$$C = M^e \bmod n$$

which is:

$$C = 10^3 \bmod 319 = 43$$

(b) Note that you and Thanaruk have an identical e in your public key. In a real system, is it less secure if two users have the same value of e? Explain your answer. [1 mark]

Answer. *No. As users have different primes, then will have different values of n. The value of e being the same is not a problem with security, and infact some values of e make improve the performance of RSA.*

Steve sent Thanaruk a confidential message. You intercepted the ciphertext, $C = 71$.

(c) What was the original plaintext, M? If an exponential is too large to perform accurately with your calculator then you may give your answer as an expression. For example, if your answer required calculating 12^{48} then you could write 12^{48} as the answer rather than trying to calculate it. Note 12^{48} is not the answer—its just an example of a value too large for your calculator. [6 marks]

Answer: ____

Answer. *If Steve sent the message confidentially to Thanaruk, then that means Steve encrypted M using Thanaruk's public key. So he performed:*

$$C = M^e \bmod n$$

which is:

$$71 = M^7 \bmod 589$$

To find M you must solve a discrete logarithm:

$$7 = dlog_{M,589}(71)$$

Although this is solvable (with a computer), using a calculator in the exam would be very hard. Lets consider another approach. We also know that:

$$M = C^d \bmod n$$

but do not know d. To find d we need to know $\phi(589)$ since d is the multiplicative inverse of $e = 7$ in $\bmod \phi(589)$. Manually finding $\phi(589)$ will take a long time in the exam, so instead factor $n = 589$ into its two prime factors. Try to divide 589 by prime numbers 3, 5, 7, 11, and so on will tell you that $31 \times 19 = 589$. That is, $p = 31$ and $q = 19$. Therefore $\phi(589) = 30 \times 18 = 540$. Now find the multiplicative inverse of 3 in $\bmod 540$. You can quickly find that:

$$3 \times 463 = 6 \times 540 + 1$$

and therefore $d = 463$. Now returning to the decryption:

$$M = 71^{463} \bmod 589$$

In the exam you can leave this as the final answer. However if you had a computer, you would find $M = 173$.

(d) Secure applications of RSA use much larger values than in the previous example. If sufficiently large values are used, then what are the three problems, all considered computationally infeasible, that an attacker must solve to break RSA? [3 marks]

Answer.

- *Factoring n into its prime factors p and q*
- *Manually determining $\phi(n)$ without knowing p and q*
- *Solving a discrete logarithm to find d directly*

Question 2 [12 marks]

(a) The one-time pad is considered to be *unconditionally secure*. What does unconditionally secure mean? [1 mark]

Answer. *Even with unlimited resource/time, the cipher is unbreakable, i.e. attacker cannot determine correct plaintext given a ciphertext.*

(b) Explain the weakness of the Vigenère cipher. [1 mark]

Answer. *For long plaintexts, repetition of the key leads to structure in the ciphertext that the attacker can take advantage of to determine the plaintext.*

(c) If a cryptanalyst knows only the encryption algorithm being used, ciphertext, and can convince the target to decrypt ciphertext values that the attack has chosen, then an attack can be classified as what type? [1 mark]

Answer. *Chosen ciphertext attack*

(d) Consider the following commands run in Linux (and assume no errors in running the commands):

```
$ echo -n "stevengordonstevengordonstevengo" > file1.txt
$ openssl enc -aes-128-cfb -in file1.txt -out file2.txt -nopad
-K f27036fbb28e554d -iv fd8a418a301fdca8
```

i. How many bits in the file `file2.txt`? [1 mark]

Answer. *256 bits. There are 32 characters, each stored as 8 bits.*

ii. How many attempts, on average, needed to perform a brute force attack on the ciphertext? [1 mark]

Answer. 2^{127}. *AES with a 128 bit key is used.*

iii. What mode of operation was used? [1 mark]

Answer. *Cipher Feedback Mode*

(e) Explain an advantage of steganography compared to encryption. [1 mark]

Answer. *With steganography, other users do not know that you are communicating something secret.*

(f) Consider a One Time Pad that uses hexadecimal (base-16) digits, as opposed to English letters. A computer system can decrypt this One Time Pad at a rate of 10^{10} messages per second. In theory, what is the worst case time to apply a brute force attack on this One Time Pad when a message is 300 characters? [1.5 marks]

Answer. $16^{300}/10^{10}$ *seconds.*

(g) Explain one approach you can use to test if a cipher exhibits the avalanche effect. In your explanation make it clear what results you expect to see if the cipher exhibits the avalanche effect. [1.5 marks]

Answer. *Method 1: Take a plaintext P and key K_1 and find the ciphertext C_1. Now modify the key by one bit to get K_2 and encrypt to get C_2. Count the number of bits which are different in the ciphertext. Repeat for different keys, and average the number of bits that differ. For the cipher to exhibit the avalanche effect, you'd expect to see the average number of bits that differ to be half of the block size. Method 2: same as method 1 but instead of modifying the key, modify the plaintext.*

(h) If you wanted to compare two encryption algorithms, A and B, with respect to the randomness of the output they produce, explain two simple tests that can be performed. [2 marks]

Answer. *Perform multiple encryptions (using different keys and plaintexts) and:*

- *Test 1. Count the number of 1's and 0's in the output. Should be equal number.*
- *Test 2. Select an M-bit block of the output and perform Test 1.*
- *Test 3. Count the length of sequences of 1's (and similar for 0's) the lengths should be small.*

Question 3 [9 marks]

(a) Encrypt the plaintext *hollow* using the Playfair cipher and keyword *steve* using the letter x as special padding if necessary. What is the ciphertext? [3 marks]

```
C = _____________________________________
```

Answer. *inkyiqxy*

The ciphertext *laizbsfavwwtwt* was obtained by encrypting using the Vigenère cipher with keyword *steve*. What was the plaintext? [3 marks]

```
P = _____________________________________
```

Answer. *theexamwaseasy*

(b) The ciphertext *dselcetistshstuhetrrafeue* was obtained by encrypting using a rail fence cipher with key *5*. What was the plaintext? [3 marks]

```
P = _____________________________________
```

Answer. *des has the feistel structure*

Question 4 [9 marks]

Consider a 4-bit block cipher, called *Steve's Simple Cipher* or SSC for short, shown in the table below. The table gives the ciphertext C produced when encrypting the plaintext P with one of the four keys.

P	C (K=00)	C (K=01)	C (K=10)	C (K=11)
0000	0110	1100	0001	0010
0001	1101	0100	1010	0000
0010	0010	0001	1111	1011
0011	0100	1101	0011	1001
0100	1100	0111	1001	0011
0101	1111	0101	0010	1000
0110	0000	0011	0111	1111
0111	0111	1011	1101	0001
1000	1010	1001	1000	0100
1001	0001	0000	1110	0111
1010	1001	0110	0110	1100
1011	1110	0010	1011	1101
1100	1011	1111	0000	0101
1101	1000	1010	0100	1110
1110	0011	1110	1100	0110
1111	0101	1000	0101	1010

(a) SSC is *not* an ideal block cipher. If SSC was to be extended to an ideal 4-bit block cipher, how many possible keys would it have? [1.5 marks]

Answer. $2^4!$

(b) If SSC was extended to be an ideal 4-bit block cipher, how long would each key be? [1.5 marks]

Answer. $4 \times 2^4 = 64$ *bits*

(c) Give a reason why ideal block ciphers are not suitable in practice. [1 mark]

Answer. *If small blocks are used, it is easy to use statistical analysis to break the cipher. With large blocks, the key length will be too long.*

Consider a block cipher, *Double-SSC*, which involves applying the block cipher SSC two times (e.g. encrypt the plaintext to obtain a temporary value, then encrypt the temporary value to obtain the ciphertext), each time using a potentially different 2-bit key.

(d) Show how the meet-in-the-middle attack works by applying it against Double-SSC. Use the attack to find the key used if the attacker already knows the (plaintext, ciphertext) pairs: (1101, 1100) and (1001, 1101). Explain clearly the steps applied by the attacker and how the key is identified. Write your answer below, and show calculations on next page. [5 marks]

`Key = ______________________`

Answer. *Considering the first pair, encrypt the plaintext with all possible values of* K_1*, and also decrypt the corresponding ciphertext with all possible values of* K_2*.*

$$P = 1101$$

$$K_{1,1} = 00 : X_{1,1} = 1000$$

$$K_{1,2} = 01 : X_{1,2} = 1010$$

$$K_{1,3} = 10 : X_{1,3} = 0100$$

$$K_{1,4} = 11 : X_{1,4} = 1110$$

$$C = 1100$$

$$K_{2,1} = 00 : X_{2,1} = 0100$$

$$K_{2,2} = 01 : X_{2,2} = 0000$$

$$K_{2,3} = 10 : X_{2,3} = 1110$$

$$K_{2,4} = 11 : X_{2,4} = 1010$$

The values of X *that match are:* $(X_{1,2}, X_{2,4})$, $(X_{1,3}, X_{2,1})$ *and* $(X_{1,4}, X_{2,3})$*. This indicates the keys are either:* $(K_{1,2} = 01, K_{2,4} = 11)$, $(K_{1,3} = 10, K_{2,1} = 00)$ *or* $(K_{1,4} = 11, K_{2,3} = 10)$*. To know which keys, then try with the second plaintext/ciphertext pair.*

$$P = 1001$$

$$K_{1,2} = 01 : X_{1,2} = 0000$$

$$X_{1,2} = 0000$$

$$K_{2,4} = 11 : C_{2,4} = 0010$$

The ciphertext obtained (`0010`) does not match the expected value (`1101`). Hence this set of keys is incorrect. Now try the next set:

$$P = 1001$$

$$K_{1,3} = 10 : X_{1,3} = 1110$$

$$X_{1,3} = 1110$$

$$K_{2,1} = 00 : C_{2,1} = 0011$$

Again, no match. That implies the third set should be correct. Lets try:

$$P = 1001$$

$$K_{1,4} = 11 : X_{1,4} = 0111$$

$$X_{1,4} = 0111$$

$$K_{2,3} = 10 : C_{2,3} = 1101$$

As expected, the ciphertext matches. Hence the keys are `11` *and* `10` *or together* `1110`.

Question 5 [7 marks]

A generalisation of the Caesar cipher is known as the *Affine Caesar cipher*. For each plaintext letter p, the ciphertext letter C is:

$$C = \mathrm{E}([a,b],p) = (ap+b) \bmod 26$$

For the Affine Caesar cipher to have a one-to-one mapping, the multiplicative inverse of a, or $\mathrm{MI}(a)$, in mod 26 must exist.

(a) Explain what is meant by a *one-to-one mapping* for a cipher. [1 mark]

Answer. *A one-to-one mapping means each input plaintext letter produces a unique ciphertext letter.*

(b) For $b = 4$ and $a > 3$, what is a value of a for which the Affine Caesar cipher has a one-to-one mapping? [1 mark]

Answer. *Any value relatively prime with 26 and greater than 3. E.g. 5, 7, 9, 11, 15, 17, 19, 21, 23, 25.*

(c) For $b = 4$ and $a > 3$, what is a value of a for which the Affine Caesar cipher does *not* have a one-to-one mapping? [1 mark]

Answer. *Any value greater than 3 and less than 26 that is not relatively prime with 26, e.g. 4, 6, 8, 10, 12, 13, 14, 16, 18, 20, 22, 24.*

(d) Using the syntax $\mathrm{MI}(a)$ for the multiplicative inverse of a, write an equation for the decryption operation of the Affine Caesar cipher. [2 marks]

Answer.

$$p = \mathrm{D}([a,b],C) = \mathrm{MI}(a)(C-b) \bmod 26$$

(e) Assume the Affine Caesar cipher is extended for an n-character alphabet, i.e. instead of mod 26 it is mod n. Write an expression that gives the number of values of a for which a one-to-one mapping exists. Explain your reasoning, i.e. why the expression is valid. [2 marks]

Answer. *For a one-to-one mapping a must have a multiplicative inverse in* mod *n. That is true if a and n are relatively prime. The number of numbers relatively prime with n (and less than n) is $\phi(n)$.*

Question 6 [7 marks]

(a) List the name of four security services desired in computer networks. For each service, explain what the service means. [4 marks]

Answer. *Confidentiality: Keep message contents secret. Authentication: Assure message and communicating parties are authenticate. Access Control: Prevent unauthorised use of a resource. Data Integrity: Assure data received are exactly as sent by authorised entity. Nonrepudiation: Protect against denial of one entity involved in communications of having participated in communications. Availability: System is accessible and usable on demand by authorised users according to intended goal.*

(b) Give the name of, and describe one active attack that can occur in computer networks. [1.5 marks]

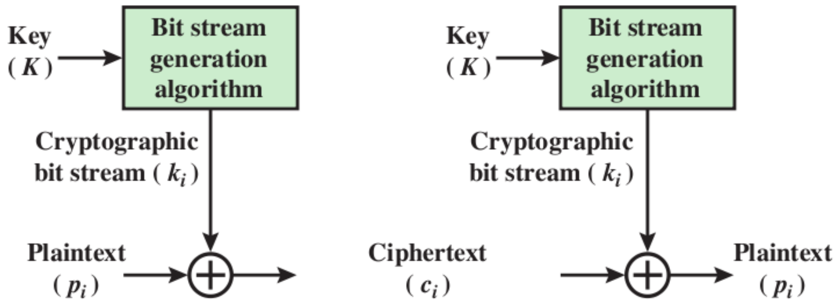
Answer. *Masquerade, denial of service, replay, modification.*

(c) Give the name of, and describe one passive attack that can occur in computer networks. [1.5 marks]

Answer. *Release message contents, traffic analysis.*

Question 7 [7 marks]

The structure of stream ciphers is shown in the following figure. The bit stream generation algorithm is a PRNG.



(a) A problem with stream ciphers is that if key K is re-used with two different plaintexts p_1 and p_2, it is relatively easy for an attacker, once they find $p_1 \oplus p_2$, to determine the original plaintext values p_1 and p_2. Explain how an attacker can find $p_1 \oplus p_2$. [3 marks]

Answer. *If the same key K is used as input, then the same keystream will be produced, k. So when encrypting p_1 the ciphertext is $c_1 = P_1 \oplus k$ and when encrypting p_2 the ciphertext is $c_2 = p_2 \oplus k$. The attacker has both ciphertexts. Using propertes of XOR:*

$$\begin{aligned} c_1 \oplus c_2 &= (p_1 \oplus k) \oplus (p_2 \oplus k) && (1) \\ &= p_1 \oplus p_2 \oplus k \oplus k && (2) \\ &= p_1 \oplus p_2 && (3) \end{aligned}$$

(b) To avoid the above problem, an initialisation vector (IV) is used with a secret to create input key K. That is, a secret key shared by the users S is chosen and combined with IV such that $K = S||IV$. While S remains the same for each plaintext encrypted, IV is incremented by 1. Explain how this avoids the above problem. [2 marks]

Answer. *Changing the IV for every encryption means a different k is produced for each plaintext. Two ciphertext values cannot be XORed to find the plaintext.*

(c) If the secret S is 100 bits in length, and IV is 20 bits in length, then explain the condition in which the attacker could still find the plaintext using the approach in part (a). [2 marks]

Answer. *As the IV is incremented and limited in length, it will eventually wrap back to an original value. This will occur after* 2^{20} *encrypted plaintext values. If the attacker can obtain two ciphertext values encrypted with the same* S *and* IV*, then they can find the plaintexts.*

Question 8 [6 marks]

The following ciphertext C was obtained by encrypting the original plaintext P with a Rows/Column Transposition cipher using a 5 digit key K_1, followed by encrypting the output of the Rows/Columns with a general Caesar cipher with key K_2. You can assume the most frequency letter in the plaintext P is 'e'. You also know the first word in the plaintext is four letters long. What is the original plaintext P?

```
C = lhszlplyueshadlletip

P = ____________________________________________________
```

(Write your answer above; perform calculations below)

Answer. *First decrypt the ciphertext using Caesar cipher. The hint that the most frequent letter in plaintext is 'e' can be used. The most frequent letter in the ciphertext is 'l' (5 occurrences), so that means 'e' encrypts to 'l' using the Caesar cipher. That means the key K_2 is 7. Then you can decrypt all letters to obtain:*

X = ealseiernxlatweexmbi

Now decrypt X using Rows/Columns. A 5 digit key means there are 5 columns. With 20 characters, there are 4 letters per column. So the columns are:

```
e e n t x
a i x w m
l e l e b
s r a e i
```

Now we need to determine the ordering of the columns such that an English phrase is constructed. Lets try different variations of the first row. We'd expect the word to start with a consonant (f or s) following by vowel or vice versa: `en? et? ex? ne? te? xe?`

Now lets try adding another vowel to those that start with a vowel, and another consonant with those that start with a consonant: `ene? ete? exe? nex? net? ten? tex? xet? xen?`

Since the first word is 4 letters long, try to make a word with another letter. You should arrive at `next`*. This gives partial ordering. Some more attempts should lead you to:*

```
n e x t e
x a m w i
l l b e e
a s i e r
```

Next exam will be easier

(space for answers)

Reference Material

S-DES operations

```
P8: 6 3 7 4 8 5 10 9  P10: 3 5 2 7 4 10 1 9 8 6
IP: 2 6 3 1 4 8 5 7  E/P: 4 1 2 3 2 3 4 1  P4: 2 4 3 1
```

$$S0 = \begin{bmatrix} 01 & 00 & 11 & 10 \\ 11 & 10 & 01 & 00 \\ 00 & 10 & 01 & 11 \\ 11 & 01 & 11 & 10 \end{bmatrix} \quad S1 = \begin{bmatrix} 00 & 01 & 10 & 11 \\ 10 & 00 & 01 & 11 \\ 11 & 00 & 01 & 00 \\ 10 & 01 & 00 & 11 \end{bmatrix}$$

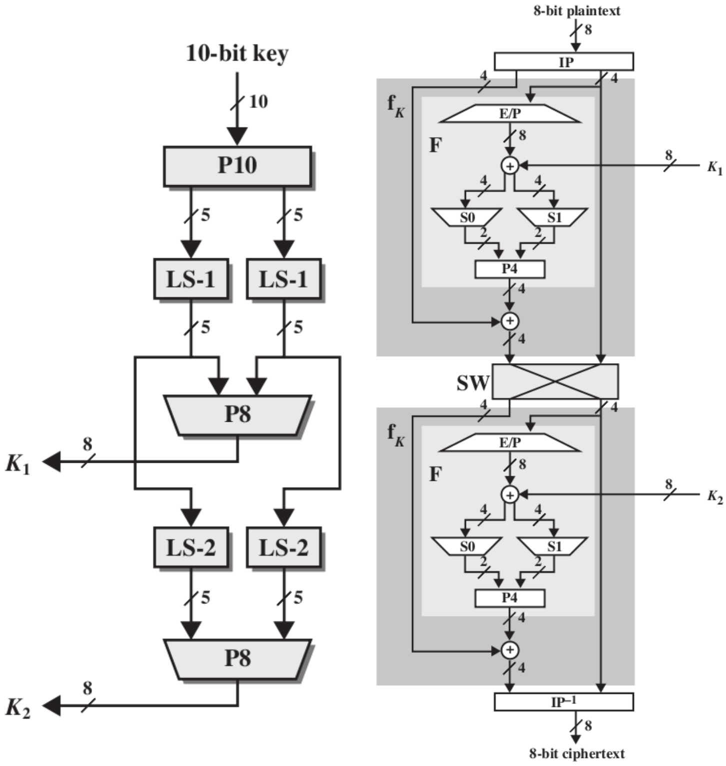


Figure 1: S-DES Key Generation and Encryption

Mapping of English characters to numbers

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Fermat's theorem if p is prime and a is a positive integer, then $a^p \equiv a \pmod{p}$

Euler's theorem For positive integers a and n, $a^{\phi(n)+1} \equiv a \pmod{n}$

First 20 prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71.

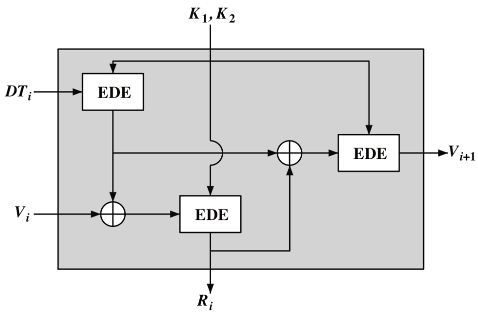
Linear Congruential Generator

$$X_{n+1} = (aX_n + c) \bmod m$$

Blum Blum Shub p, q are large prime numbers such that $p \equiv q \equiv 3 \pmod 4$; $n = p \times q$; s, random number relatively prime to n. Generate sequence of bits, B_i:

$$\begin{aligned} X_0 &= s^2 \bmod n \\ \text{for } i &= 1 \rightarrow \infty \\ X_i &= (X_{i-1})^2 \bmod n \\ B_i &= X_i \bmod 2 \end{aligned}$$

ANSI X9.17 See figure below:



Modes of operation

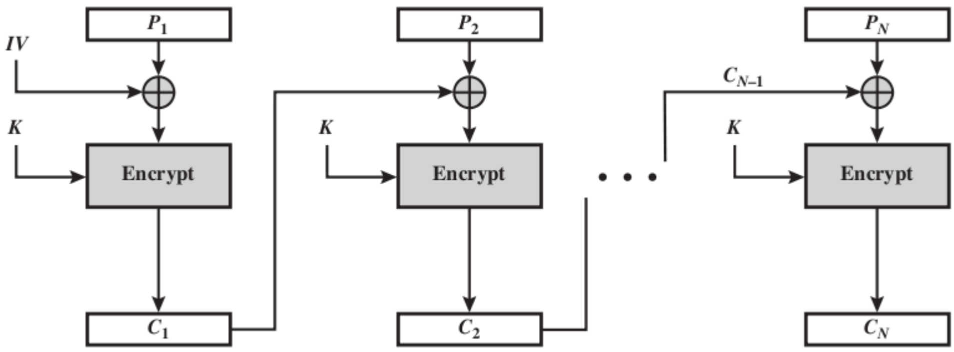


Figure 2: CBC mode of operation

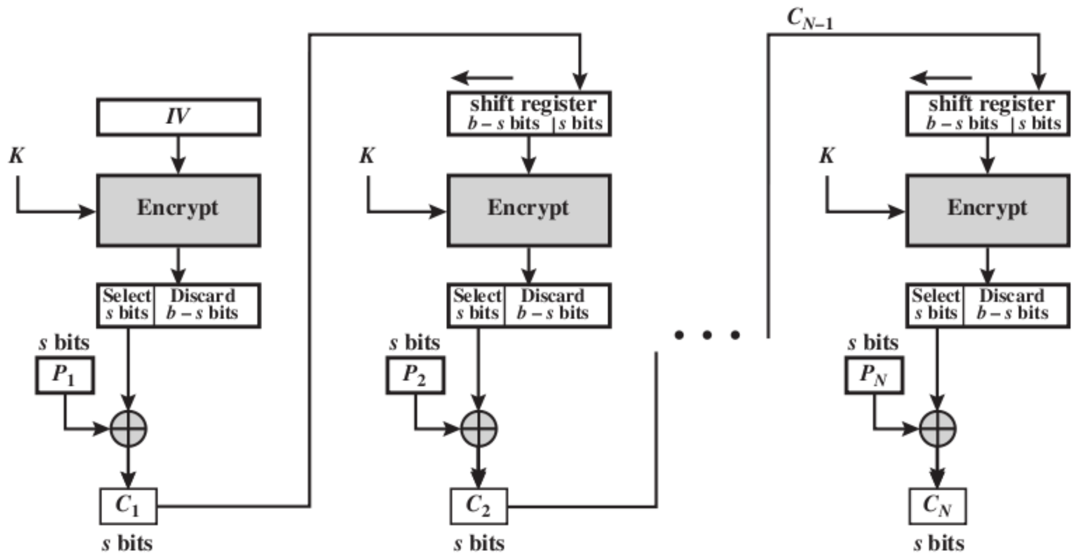


Figure 3: CFB mode of operation

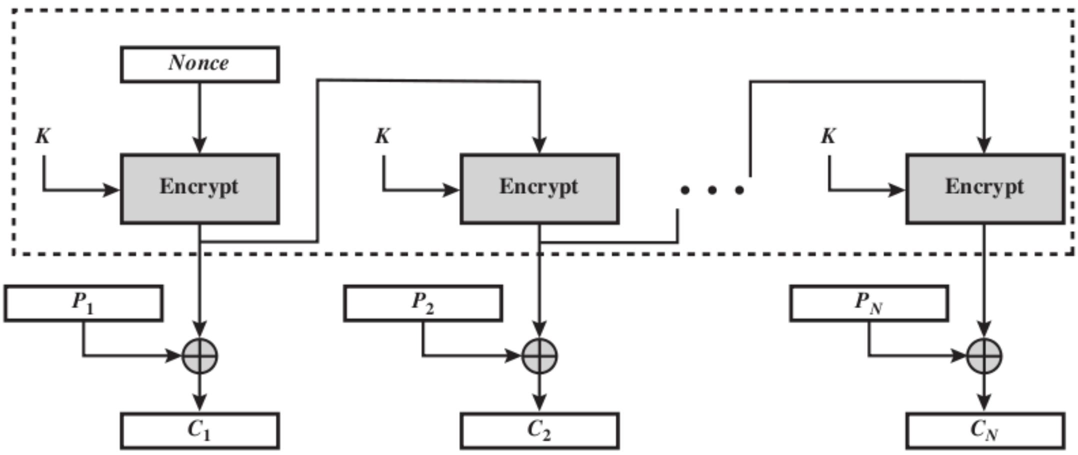


Figure 4: OFB mode of operation

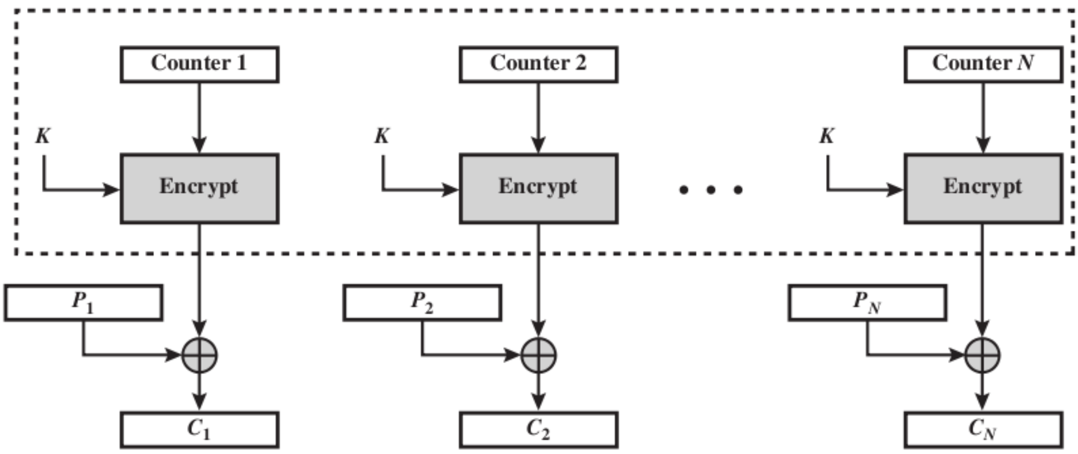


Figure 5: CTR mode of operation