Pseudo-Random Numbers and Stream Ciphers

CSS322: Security and Cryptography

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css322y13s2l05, Steve/Courses/2013/s2/css322/lectures/random.tex, r2965

Contents

Random Numbers

Use of Random Numbers

- Key distribution and authentication schemes
- Generation of session keys or keys for RSA
- Generation of bit stream for stream ciphers

Randomness

- Uniform distribution: frequency of occurrence of 1's and 0's approximately equal
- Independence: no sub-sequence can be inferred from others

Unpredictability

- Hard to predict next value in sequence

TRNG, PRNG and PRF

True Random Number Generator

- Non-deterministic source, physical environment
- Detect ionizing radiation events, leaky capacitors, thermal noise from resistors or audio inputs
- Mouse/keyboard activity, I/O operations, interrupts
- Inconvenient, small number of values

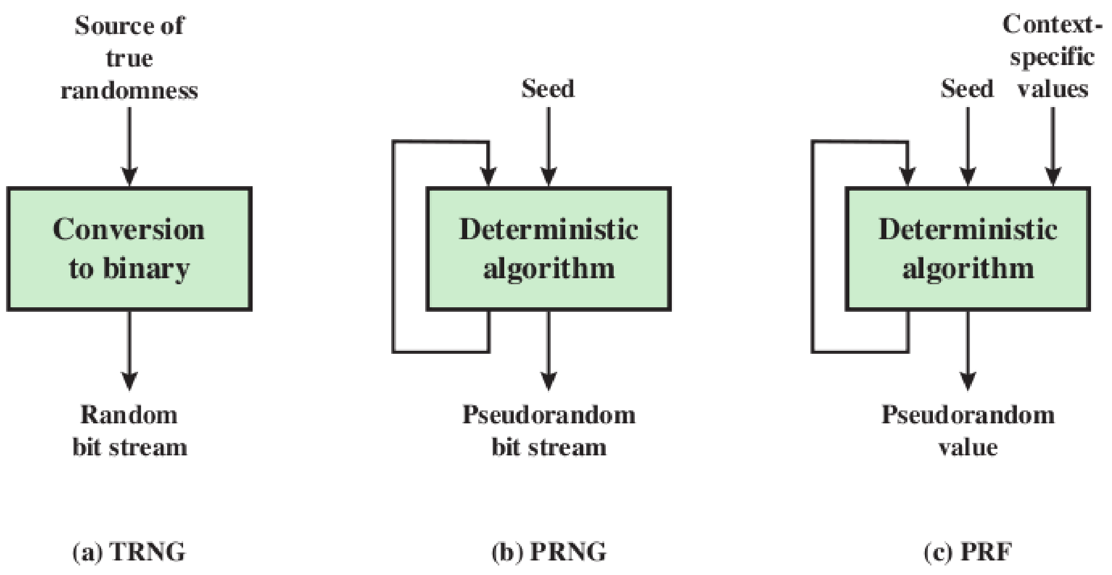
Pseudo Random Number Generator

- Deterministic algorithms to calculate numbers in "relatively random" sequence
- Seed is algorithm input
- Produces continuous stream of random bits

Pseudo Random Function

- Same as PRNG but produces string of bits of some

Random and Pseudo-Random Number Generators

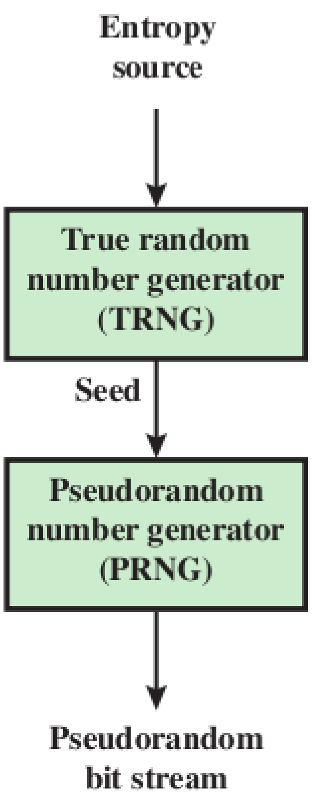


Requirements of PRNG

Hard to determine pseudo-random stream if don't know seed (but know algorithm)

- Randomness
  - Test for uniformity, scalability, consistency
  - Examples: Frequency, runs, compressability
- Unpredictability
  - Forward and backward unpredictability
- Seed must be secure
  - Use TRNG to generate seed

Generation of Seed Input to PRNG


Entropy source
True random number generator (TRNG)
Seed
Pseudorandom number generator (PRNG)
Pseudorandom bit stream

Contents

Linear Congruential Generator

Parameters:

- m, the modulus, $m > 0$
- a, the multiplier, $0 < a < m$
- c, the increment, $0 \leq c < m$
- X_0, the seed, $0 \leq X_0 < m$

Generate sequence of pseudo-random numbers, $\{X_n\}$:

$$X_{n+1} = (aX_n + c) \bmod m$$

Choice of a, c and m is important:

- m should be large, prime, e.g. $2^{31} - 1$
- If c=0, few good values of a, e.g. $7^5 = 16807$

If attacker knows parameters and one number, can easily determine subsequent numbers

Blum Blum Shub Generator

Parameters:

- p, q: large prime numbers such that $p \equiv q \equiv 3 \pmod 4$
- $n = p \times q$
- s, random number relatively prime to n

Generate sequence of bits, B_i:

$$\begin{aligned} X_0 &= s^2 \bmod n \\ \text{for } i &= 1 \rightarrow \infty \\ X_i &= (X_{i-1})^2 \bmod n \\ B_i &= X_i \bmod 2 \end{aligned}$$

Cryptographically secure pseudo-random bit generator

Example Operation of BBS Generator

$n = 192649 = 383 \times 503$, $s = 101355$

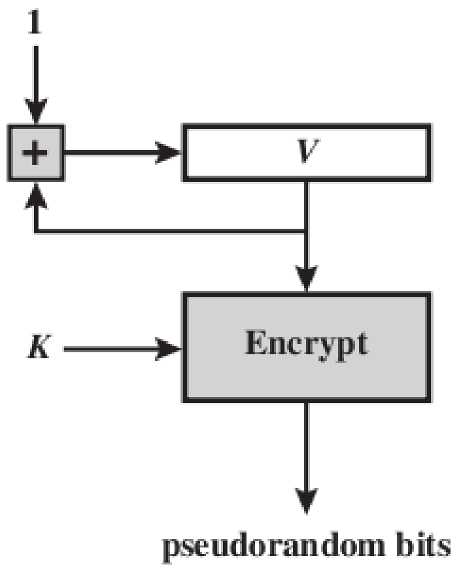
i	X_i	B_i
0	20749	
1	143135	1
2	177671	1
3	97048	0
4	89992	0
5	174051	1
6	80649	1
7	45663	1
8	69442	0
9	186894	0
10	177046	0
11	137922	0
12	123175	1
13	8630	0
14	114386	0
15	14863	1
16	133015	1
17	106065	1
18	45870	0
19	137171	1
20	48060	0

Contents

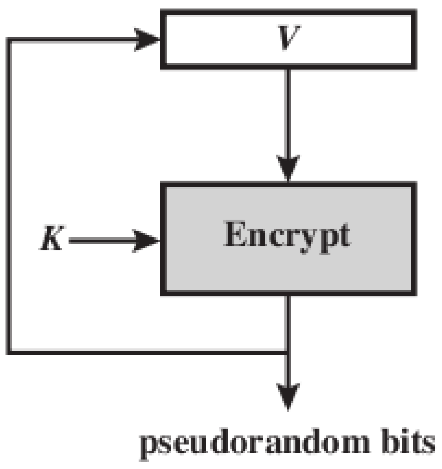
PRNG Mechanisms Based on Block Ciphers

Use symmetric block ciphers (e.g. AES, DES) to produce pseudo-random bits

- Seed is encryption key, K, and value V (which is updated)



Counter Mode



OFB Mode

ANSI X9.17 PRNG

Cryptographically secure PRNG using Triple DES

Parameters:

- 64-bit date/time representation, DT_i
- 64-bit seed value, V_i
- Pair of 56-bit DES keys, K_1 and K_2

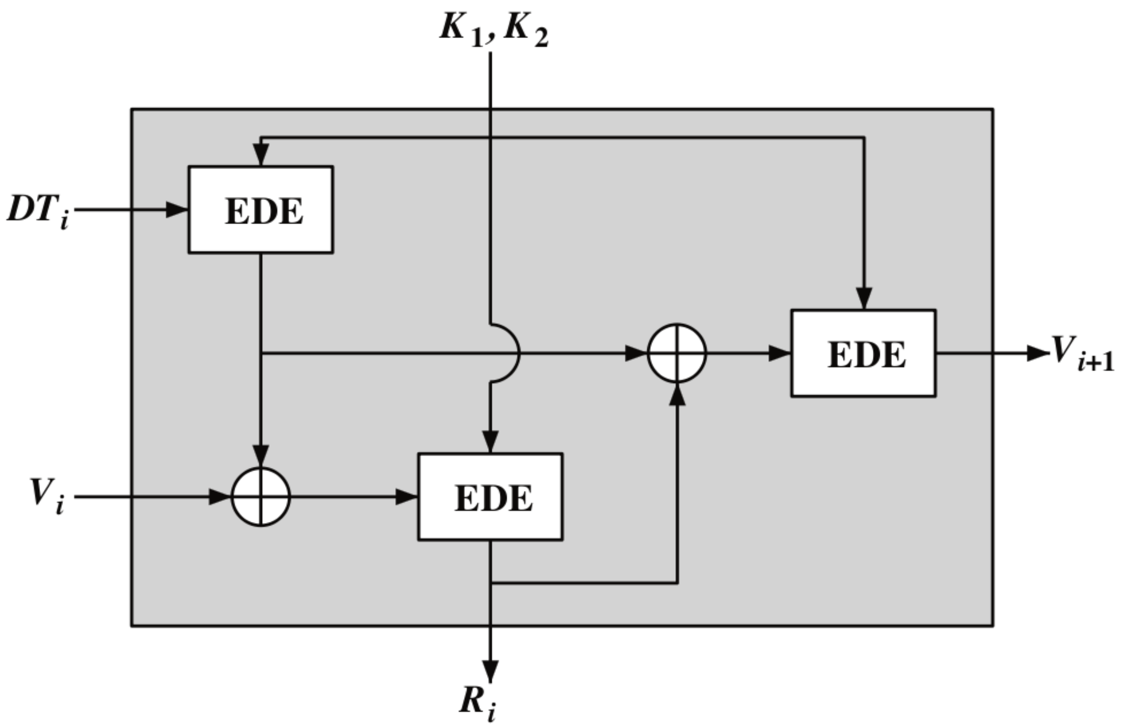
Operation:

- Uses Triple DES three times
- (see next slide)

Output:

- 64-bit pseudo-random number, R_i
- 64-bit seed value, V_{i+1}

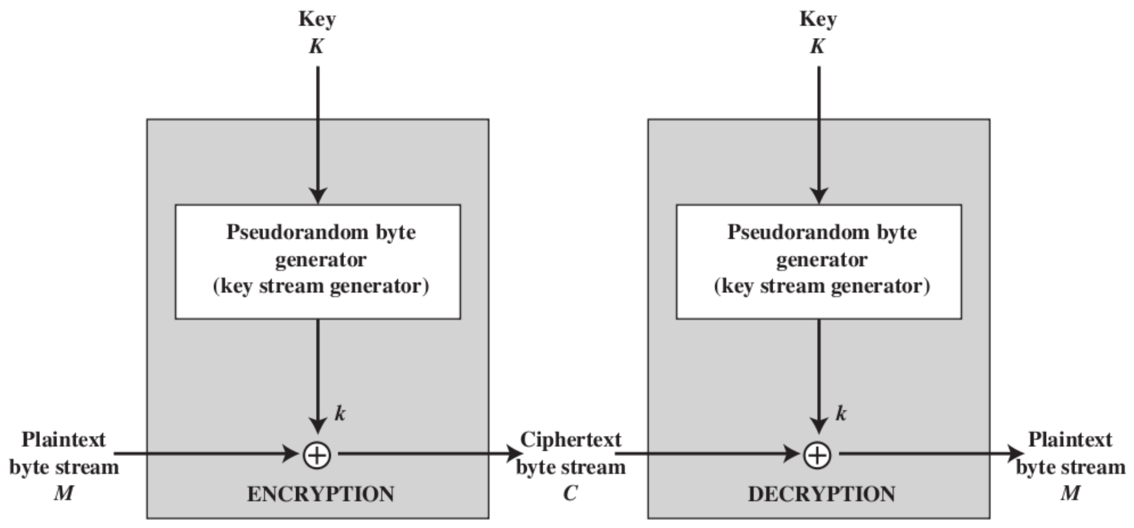
ANSI X9.17 PRNG



Contents

Stream Ciphers

Encrypt one byte at a time by XOR with pseudo-random byte


Key
K
Pseudorandom byte generator (key stream generator)
k
Plaintext byte stream M
ENCRYPTION
Ciphertext byte stream C
Key
K
Pseudorandom byte generator (key stream generator)
k
DECRYPTION
Plaintext byte stream M

Output of generator is called keystream

Design Criteria for Stream Ciphers

Important Considerations

- Encryption sequence should have large period
- Keystream should approximate true random number stream
- Key must withstand brute force attacks

Comparison to Block Ciphers

- Stream ciphers often simpler to implement, faster
- Block ciphers can re-use keys

Contents

RC4

- Designed by Ron Rivest in 1987
- Used in secure web browsing and wireless LANs
- Very simple and efficient implementation
- Can use variable size key: 8 to 2048 bits
- Several theoretical limitations of RC4
  - No known attacks if use 128-bit key and discard initial values of stream
  - RC4 is used in WEP (shown to be weak security for wireless LANs)—problem with how keys are used, not RC4 algorithm

RC4 Algorithm

Parameters and Variables

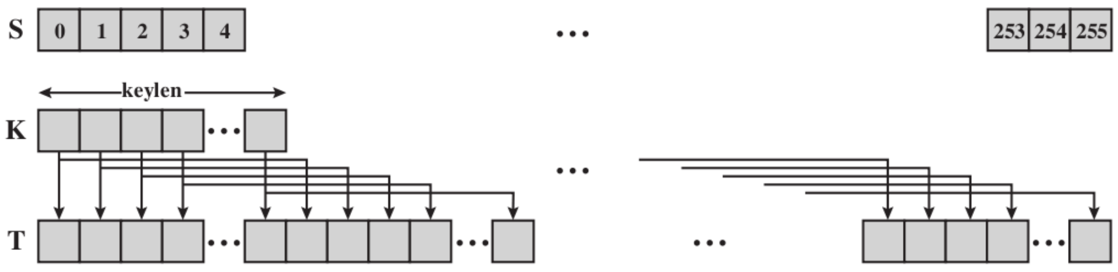
- Variable length key, K, from 1 to 256 Bytes
- State vector, S, 256 Bytes
- Temporary vector, T, 256 Bytes
- A byte from keystream, k, generated from S

Steps

1. Initialise S to values 0 to 255; initialise T with repeating values of key, K
2. Use T to create initial permutation of S
3. Permutate S and generate keystream, k from S
4. Encrypt a byte of plaintext, p, by XOR with k

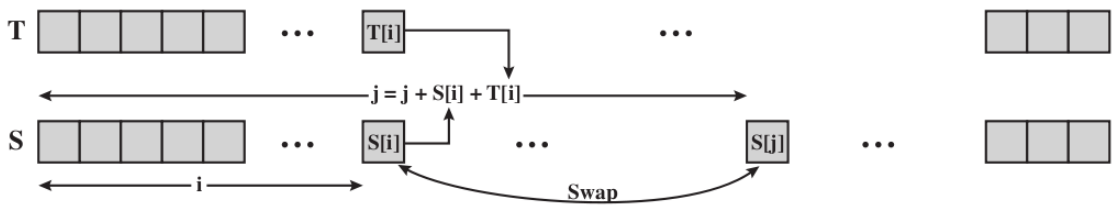
Initial State of S and T

```
for i = 0 to 255 do
   S[i] = i;
   T[i] = K[i mod keylen];
```



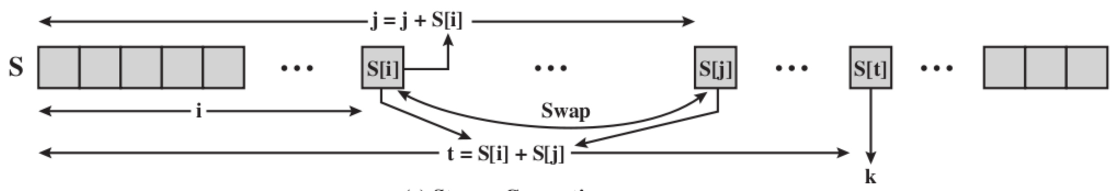
Initial Permutation of S

```
j = 0;
for i = 0 to 255 do
   j = (j + S[i] + T[i]) mod 256;
Swap (S[i], S[j]);
```



Stream Generation

```
i, j = 0;
while (true)
   i = (i + 1) mod 256;
   j = (J + S[i]) mod 256;
   Swap (S[i], S[j]);
   t = (S[i] + S[j]) mod 256;
   k = S[t];
```



To encrypt: $C = p \text{ XOR } k$

To decrypt: $p = C \text{ XOR } k$